PROBLEMS OF COUPLED THERMOELASTICITY FOR A PARALLELEPIPED

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We present the solution of quastatic problems of coupled thermoelasticity for a parallelepiped.

General coupled problems in the mechanics of continuous media were formulated in [1]. The problems of coupled thermoelasticity were reviewed in [2-5]. For a linear isotropic homogeneous thermoelastic medium under small deformations, the balance and heat-flux equations have the form [5]

$$\mu\Delta u_i + (\lambda + \mu) e_{,i} + X_i = \gamma\theta_{,i} \quad (i = 1,\ 2,\ 3), \tag{1}$$

$$\Delta\theta - \frac{1}{\varkappa}\dot{\theta} - \eta\dot{e} + Q = 0, \tag{2}$$

$$e = u_{k,k},\quad \varkappa = \frac{\lambda^0}{c_\varepsilon},\quad \eta = \frac{\gamma T_0}{\lambda^0},\quad Q = \frac{w}{\lambda^0},\quad \gamma = (3\lambda + 2\mu)\alpha_0.$$

Since the case of arbitrary volume forces always reduces to the case when the volume forces have a potential [7], we shall assume that $X_i = G_{,i}$. The initial conditions are

$$u_i|_{t=0} = f_i(x_k),\quad \theta|_{t=0} = h(x_k). \tag{3}$$

The solution of the quasistatic problem of coupled thermoelasticity consists of the integration of the system of equations (1) and (2) subject to the initial conditions (3) and some boundary conditions for u_i and θ which will not be specified at the moment.

Integrating (2) with respect to time from 0 to t and using (3), we obtain

$$\frac{\theta}{\varkappa} = \int_0^t \Delta\theta d\tau - \eta e + L \left(L \equiv \frac{h}{\varkappa} + \eta f_{k,k} + \int_0^t Q d\tau\right). \tag{4}$$

Substituting the expression (4) for θ into (1) we find

$$\mu\Delta u_i + (\lambda_1 + \mu) e_{,i} = \gamma\varkappa \int_0^t (\Delta\theta)_{,i}\, d\tau + (\gamma\varkappa L - G)_{,i}, \tag{5}$$

where $\lambda_1 = \lambda + \eta\gamma\varkappa$. We shall write u_i in the form $u_i = v_i + w_i$, $e = e_v + e_w$, $e_v = v_{k,k}$, $e_w = w_{k,k}$, where v_i is the solution of the system

$$\mu\Delta v_i + (\lambda_1 + \mu) e_{v,i} = 0, \tag{6}$$

and w_i is an arbitrary particular solution of the system (5). We shall seek w_i in the form $w_i = \Phi_{,i}$. Substituting this expression into (5) and noting that

$$\Delta(\operatorname{grad}\Phi) = \operatorname{grad}\operatorname{div}(\operatorname{grad}\Phi) - \operatorname{rot}\operatorname{rot}(\operatorname{grad}\Phi) = \operatorname{grad}\Delta\Phi,$$

we find Φ:

$$\Phi = \gamma_1 \int_0^t \theta d\tau + F,\quad \gamma_1 \equiv \frac{\gamma\varkappa}{\lambda_1 + 2\mu},$$

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where F is an arbitrary particular solution of the equation $(\lambda_i + 2\mu)\Delta F = \gamma \varkappa L - G$. Therefore,

$$u_i = v_i + \gamma_1 \int_0^t \theta_{,i} d\tau + F_{,i}. \tag{7}$$

It follows from (7) that

$$e = e_v + \gamma_1 \int_0^t \Delta\theta d\tau + \Delta F. \tag{8}$$

Substituting (8) into (4), we obtain

$$\theta - k^2 \int_0^t \Delta\theta d\tau = M - \eta\varkappa e_v, \tag{9}$$

$$(\lambda_1 + 2\mu) k^2 = \varkappa(\lambda + 2\mu), \quad (\lambda_1 + 2\mu) M = \varkappa[(\lambda + 2\mu) L + \eta G].$$

Eliminating $\int_0^t \Delta\theta d\tau$ from (8) and (9) we find the simplest expression for e:

$$(\lambda + 2\mu) e = (\lambda_1 + 2\mu) e_v + \gamma\theta - G. \tag{10}$$

It follows from (9) that

$$\dot{\theta} - k^2\Delta\theta = \dot{M} - \eta\varkappa\dot{e}_v. \tag{11}$$

In some problems v_i, and consequently also e_v, can be found without using Eq. (11). By substituting the value of e_v into (11) we then obtain an equation for θ which has a known right-hand side and is supplemented by some boundary condition. Having found θ, we can determine u_i from Eq. (7).

In an orthogonal Cartesian coordinate system $Ox_1x_2x_3$, the components of the stress tensor are

$$\sigma_{11} = 2\mu \frac{\partial u_1}{\partial x_1} + \lambda e - \gamma\theta = (\lambda + 2\mu) e - \gamma\theta - 2\mu \left(\frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right), \tag{12}$$

$$\sigma_{22} = 2\mu \frac{\partial u_2}{\partial x_2} + \lambda e - \gamma\theta, \quad \sigma_{33} = 2\mu \frac{\partial u_3}{\partial x_3} + \lambda e - \gamma\theta, \tag{13}$$

$$\sigma_{12} = \mu \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right), \quad \sigma_{13} = \mu \left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right), \quad \sigma_{23} = \mu \left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right). \tag{14}$$

The system (6) will be written in the form

$$\varkappa_1 \frac{\partial e_v}{\partial x_1} = \frac{\partial \omega_{v3}}{\partial x_2} - \frac{\partial \omega_{v2}}{\partial x_3}, \quad \varkappa_1 \frac{\partial e_v}{\partial x_2} = \frac{\partial \omega_{v1}}{\partial x_3} - \frac{\partial \omega_{v3}}{\partial x_1}, \tag{15}$$

$$\varkappa_1 \frac{\partial e_v}{\partial x_3} = \frac{\partial \omega_{v2}}{\partial x_1} - \frac{\partial \omega_{v1}}{\partial x_2}, \quad \left(\varkappa_1 \equiv 2 + \frac{\lambda_1}{\mu}\right), \tag{16}$$

$$e_v = \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3}, \quad \omega_{v1} = \frac{\partial v_3}{\partial x_2} - \frac{\partial v_2}{\partial x_3}, \tag{17}$$

$$\omega_{v2} = \frac{\partial v_1}{\partial x_3} - \frac{\partial v_3}{\partial x_1}, \quad \omega_{v3} = \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}. \tag{18}$$

Suppose now we are given a parallelepiped, and $x_1 = 0$, $x_1 = a$, $x_2 = 0$, $x_2 = b$, $x_3 = 0$, $x_3 = c$ are the equations of its sides. By using the methods explained below we can find the solution of the problems where one is given normal displacements, tangential stresses, and the heat flux at any k sides (k = 0, 1, 2, ..., 6) of the parallelepiped, and on the remaining 6 − k sides one is given tangential displacements, normal stresses, and the temperature. However, the consideration of all possible variants would make this paper excessively long. We shall therefore consider only the case k = 6. The solution of this problem will be obtained in the form of a sum of solutions of the simpler problems.

Problem 1. Normal displacements, tangential stresses, and the heat flux are given at all sides of the parallelepiped:

$$u_1|_{x_1=0} = \delta_1, \quad \sigma_{12}|_{x_1=0} = S_1, \quad \sigma_{13}|_{x_1=0} = T_1, \quad \left.\frac{\partial\theta}{\partial x_1}\right|_{x_1=0} = q_1, \tag{19}$$

$$u_1|_{x_1=a} = \delta_2, \quad \sigma_{12}|_{x_1=a} = S_2, \quad \sigma_{13}|_{x_1=a} = T_2, \quad \left.\frac{\partial\theta}{\partial x_1}\right|_{x_1=a} = q_2, \tag{20}$$

$$u_2|_{x_2=0} = \delta_3, \quad \sigma_{12}|_{x_2=0} = S_3, \quad \sigma_{23}|_{x_2=0} = T_3, \quad \left.\frac{\partial\theta}{\partial x_2}\right|_{x_2=0} = q_3, \tag{21}$$

$$u_2|_{x_2=b} = \delta_4, \quad \sigma_{12}|_{x_2=b} = S_4, \quad \sigma_{23}|_{x_2=b} = T_4, \quad \left.\frac{\partial\theta}{\partial x_2}\right|_{x_2=b} = q_4, \tag{22}$$

$$u_3|_{x_3=0} = \delta_5, \quad \sigma_{13}|_{x_3=0} = S_5, \quad \sigma_{23}|_{x_3=0} = T_5, \quad \left.\frac{\partial\theta}{\partial x_3}\right|_{x_3=0} = q_5, \tag{23}$$

$$u_3|_{x_3=c} = \delta_6, \quad \sigma_{23}|_{x_3=c} = S_6, \quad \sigma_{23}|_{x_3=c} = T_6, \quad \left.\frac{\partial\theta}{\partial x_3}\right|_{x_3=c} = q_6. \tag{24}$$

The following expressions will be useful in the discussion below:

$$u_1|_{x_1=0} = 0, \quad \sigma_{12}|_{x_1=0} = 0, \quad \sigma_{13}|_{x_1=0} = 0, \quad \left.\frac{\partial\theta}{\partial x_1}\right|_{x_1=0} = 0, \tag{25}$$

$$u_1|_{x_1=a} = 0, \quad \sigma_{12}|_{x_1=a} = 0, \quad \sigma_{13}|_{x_1=a} = 0, \quad \left.\frac{\partial\theta}{\partial x_1}\right|_{x_1=a} = 0, \tag{26}$$

$$u_2|_{x_2=0} = 0, \quad \sigma_{12}|_{x_2=0} = 0, \quad \sigma_{23}|_{x_2=0} = 0, \quad \left.\frac{\partial\theta}{\partial x_2}\right|_{x_2=0} = 0, \tag{27}$$

$$u_2|_{x_2=b} = 0, \quad \sigma_{12}|_{x_2=b} = 0, \quad \sigma_{23}|_{x_2=b} = 0, \quad \left.\frac{\partial\theta}{\partial x_2}\right|_{x_2=b} = 0, \tag{28}$$

$$u_3|_{x_3=0} = 0, \quad \sigma_{13}|_{x_3=0} = 0, \quad \sigma_{23}|_{x_3=0} = 0, \quad \left.\frac{\partial\theta}{\partial x_3}\right|_{x_3=0} = 0, \tag{29}$$

$$u_3|_{x_3=c} = 0, \quad \sigma_{13}|_{x_3=c} = 0, \quad \sigma_{23}|_{x_3=c} = 0, \quad \left.\frac{\partial\theta}{\partial x_3}\right|_{x_3=c} = 0. \tag{30}$$

Problem 1a. Let us suppose that the boundary conditions have the form (19), (20), and (27)-(30). The unknown functions will be expanded in Fourier series as follows:

$$u_1 = \sum_{m,n=0}^{+\infty} u_{1mn}(x_1, t)\cos\alpha_m x_2 \cos\beta_n x_3, \quad u_2 = \sum_{m,n=0}^{+\infty} u_{2mn}(x_1, t)\sin\alpha_m x_2 \cos\beta_n x_3,$$

$$u_3 = \sum_{m,n=0}^{+\infty} u_{3mn}(x_1, t)\cos\alpha_m x_2 \sin\beta_n x_3,$$

$$\theta = \sum_{m,n=0}^{+\infty} \theta_{mn}(x_1, t)\cos\alpha_m x_2 \cos\beta_n x_3,$$

where $\alpha_m \equiv \pi m/b$ and $\beta_n \equiv \pi n/c$. The expansions of other functions can easily be written down, and will not be given here. The corresponding expansion coefficients will be denoted by indices m and n. In the expressions below, repeating indices m and n are not summed over. The conditions (27)-(30) are satisfied. Using (10) in (12) we obtain, instead of (12)-(18), for each m and n

$$\sigma_{11mn} = (\lambda_1 + 2\mu)\, e_{vmn} - G_{mn} - 2\mu(\alpha_m u_{2mn} + \beta_n u_{3mn}), \tag{31}$$

$$\sigma_{22mn} = 2\mu\alpha_m u_{2mn} + \lambda e_{mn} - \gamma\theta_{mn}, \quad \sigma_{33mn} = 2\mu\beta_n u_{3mn} + \lambda e_{mn} - \gamma\theta_{mn},$$

$$\sigma_{12mn} = \mu\left(\frac{\partial u_{2mn}}{\partial x_1} - \alpha_m u_{1mn}\right), \quad \sigma_{13mn} = \mu\left(\frac{\partial u_{3mn}}{\partial x_1} - \beta_n u_{1mn}\right), \tag{32}$$

$$\sigma_{23mn} = -\mu(\beta_n u_{2mn} + \alpha_m u_{3mn}),$$

$$\varkappa_1 \frac{\partial e_{vmn}}{\partial x_1} = \Omega_{vmn}, \quad (\Omega_{vmn} \equiv \alpha_m \omega_{v3mn} - \beta_n \omega_{v2mn}), \tag{33}$$

$$\varkappa_1 \alpha_m e_{vmn} = \frac{\partial \omega_{v3mn}}{\partial x_1} - \beta_n \omega_{v1mn}, \tag{34}$$

$$\varkappa_1 \beta_n e_{vmn} = \alpha_m \omega_{v1mn} - \frac{\partial \omega_{v2mn}}{\partial x_1}, \tag{35}$$

$$e_{vmn} = \frac{\partial v_{1mn}}{\partial x_1} + \alpha_m v_{2mn} + \beta_n v_{3mn}, \quad \omega_{v1mn} = \beta_n v_{2mn} - \alpha_m v_{3mn}, \tag{36}$$

$$\omega_{v2mn} = -\frac{\partial v_{3mn}}{\partial x_1} - \beta_n v_{1mn}, \quad \omega_{v3mn} = \frac{\partial v_{2mn}}{\partial x_1} + \alpha_m v_{1mn}. \tag{37}$$

Relations (7), (11), and (3) give

$$u_{1mn} = v_{1mn} + \gamma_1 \int_0^t \frac{\partial \theta_{mn}}{\partial x_1} d\tau + \frac{\partial F_{mn}}{\partial x_1}, \tag{38}$$

$$u_{2mn} = v_{2mn} - \gamma_1 \alpha_m \int_0^t \theta_{mn} d\tau - \alpha_m F_{mn}, \tag{39}$$

$$u_{3mn} = v_{3mn} - \gamma_1 \beta_n \int_0^t \theta_{mn} d\tau - \beta_n F_{mn}, \tag{40}$$

$$\dot{\theta}_{mn} - k^2 \left(\frac{\partial^2 \theta_{mn}}{\partial x_1^2} - \lambda_{mn}^2 \theta_{mn} \right) = \dot{M}_{mn} - \eta \varkappa \dot{e}_{vmn}, \quad \theta_{mn}|_{t=0} = h_{mn}, \tag{41}$$

where $\lambda_{mn}^2 \equiv \alpha_m^2 + \beta_n^2$. Multiplying (34) and (35) by α_m and β_n, respectively, and adding, we obtain

$$\varkappa_1 \lambda_{mn}^2 e_{vmn} = \frac{\partial \Omega_{vmn}}{\partial x_1}. \tag{42}$$

Substituting Ω_{vmn} from (33) into (42), we find

$$e_{vmn} = c_{1mn}(t) \exp(-\lambda_{mn} x_1) + c_{2mn}(t) \exp(\lambda_{mn} x_1), \tag{43}$$

$$\Omega_{vmn} = \varkappa_1 \lambda_{mn} [-c_{1mn}(t) \exp(-\lambda_{mn} x_1) + c_{2mn}(t) \exp(\lambda_{mn} x_1)]. \tag{44}$$

Expressing Ω_{vmn} in terms of the displacements by using (37), we obtain

$$e_{vmn} = \frac{\partial v_{1mn}}{\partial x_1} + R_{mn}, \quad \Omega_{vmn} = \lambda_{mn}^2 v_{1mn} + \frac{\partial R_{mn}}{\partial x_1}, \tag{45}$$

where $R_{mn} \equiv \alpha_m v_{2mn} + \beta_n v_{3mn}$. Therefore,

$$v_{1mn} = \frac{1}{\lambda_{mn}^2} \left(\Omega_{vmn} - \frac{\partial R_{mn}}{\partial x_1} \right), \quad \frac{\partial^2 R_{mn}}{\partial x_1^2} - \lambda_{mn}^2 R_{mn} = \frac{\partial \Omega_{vmn}}{\partial x_1} - \lambda_{mn}^2 e_{vmn}.$$

Hence, using (43) and (44),

$$\begin{aligned} R_{mn} = {} & \lambda_{mn} \varkappa_2 x_1 [c_{1mn}(t) \exp(-\lambda_{mn} x_1) - c_{2mn}(t) \exp(\lambda_{mn} x_1)] + \\ & + c_{3mn}(t) \exp(-\lambda_{mn} x_1) + c_{4mn}(t) \exp(\lambda_{mn} x_1), \quad (2\varkappa_2 \equiv 1 - \varkappa_1), \end{aligned} \tag{46}$$

$$\begin{aligned} v_{1mn}(x_1, t) = {} & (\varkappa_2 x_1 - \varkappa_3) c_{1mn}(t) \exp(-\lambda_{mn} x_1) + (\varkappa_2 x_1 + \varkappa_3) c_{2mn}(t) \times \\ & \times \exp(\lambda_{mn} x_1) + \frac{1}{\lambda_{mn}} c_{3mn}(t) \exp(-\lambda_{mn} x_1) - \frac{1}{\lambda_{mn}} c_{4mn}(t) \exp(\lambda_{mn} x_1), \\ & (2\lambda_{mn} \varkappa_3 \equiv 1 + \varkappa_1). \end{aligned} \tag{47}$$

Multiplying (34) and (35) by β_n and α_m, respectively, and subtracting term by term, we find

$$\omega_{v1mn} = \frac{1}{\lambda_{mn}^2}\left(\alpha_m \frac{\partial \omega_{v2mn}}{\partial x_1} + \beta_n \frac{\partial \omega_{v3mn}}{\partial x_1}\right). \tag{48}$$

Substituting (37) into (48), we obtain

$$\frac{\partial^2 \omega_{v1mn}}{\partial x_1^2} - \lambda_{mn}^2 \omega_{v1mn} = 0, \qquad \omega_{v1mn} = c_{5mn}(t)\exp(-\lambda_{mn}x_1) + c_{6mn}(t)\exp(\lambda_{mn}x_1). \tag{49}$$

Since $\alpha_m v_{2mn} + \beta_n v_{3mn} = R_{mn}$ and $\beta_n v_{2mn} - \alpha_m v_{3mn} = \omega_{v1mn}$ we find, by using (46) and (49),

$$\lambda_{mn}^2 v_{2mn} = \alpha_m \lambda_{mn} \varkappa_2 x_1 [c_{1mn}(t)\exp(-\lambda_{mn}x_1) - c_{2mn}(t)\exp(\lambda_{mn}x_1)] + \\ + \alpha_m [c_{3mn}(t)\exp(-\lambda_{mn}x_1) + c_{4mn}(t)\exp(\lambda_{mn}x_1)] + \beta_n [c_{5mn}(t)\exp(-\lambda_{mn}x_1) + c_{6mn}(t)\exp(\lambda_{mn}x_1)], \tag{50}$$

$$\lambda_{mn}^2 v_{3mn} = \beta_n \lambda_{mn} \varkappa_2 x_1 [c_{1mn}(t)\exp(-\lambda_{mn}x_1) - c_{2mn}(t)\exp(\lambda_{mn}x_1)] + \\ + \beta_n [c_{3mn}(t)\exp(-\lambda_{mn}x_1) + c_{4mn}(t)\exp(\lambda_{mn}x_1)] - \alpha_m [c_{5mn}(t)\exp(-\lambda_{mn}x_1) + c_{6mn}(t)\exp(\lambda_{mn}x_1)]. \tag{51}$$

Using (32) we obtain for each m and n, instead of (19) and (20),

$$u_{1mn}|_{x_1=0} = \delta_{1mn}(t), \quad u_{1mn}|_{x_1=a} = \delta_{2mn}(t), \tag{52}$$

$$\mu\left(\frac{\partial u_{2mn}}{\partial x_1} - \alpha_m u_{1mn}\right)\Bigg|_{x_1=0} = S_{1mn}(t), \quad \mu\left(\frac{\partial u_{3mn}}{\partial x_1} - \beta_n u_{1mn}\right)\Bigg|_{x_1=0} = T_{1mn}(t), \tag{53}$$

$$\mu\left(\frac{\partial u_{2mn}}{\partial x_1} - \alpha_m u_{1mn}\right)\Bigg|_{x_1=a} = S_{2mn}(t), \quad \mu\left(\frac{\partial u_{3mn}}{\partial x_1} - \beta_n u_{1mn}\right)\Bigg|_{x_1=a} = T_{2mn}(t), \tag{54}$$

$$\frac{\partial \theta_{mn}}{\partial x_1}\Bigg|_{x_1=0} = q_{1mn}(t), \quad \frac{\partial \theta_{mn}}{\partial x_1}\Bigg|_{x_1=a} = q_{2mn}(t). \tag{55}$$

By a direct calculation based on (39) and (40) we obtain

$$\beta_n u_{2mn} - \alpha_m u_{3mn} = \beta_n v_{2mn} - \alpha_m v_{3mn} = \omega_{v1mn}, \tag{56}$$

and hence we obtain, using (52)-(54),

$$\mu \frac{\partial \omega_{v1mn}}{\partial x_1}\Bigg|_{x_1=0} = \beta_n S_{1mn} - \alpha_m T_{1mn}; \quad \mu \frac{\partial \omega_{v1mn}}{\partial x_1}\Bigg|_{x_1=a} = \beta_n S_{2mn} - \alpha_m T_{2mn}. \tag{57}$$

Substituting (49) into (57) we find $c_{5mn}(t)$ and $c_{6mn}(t)$. A direct calculation using (38)-(40) and (45) then gives

$$\lambda_{mn}^2 u_{1mn} + \frac{\partial}{\partial x_1}(\alpha_m u_{2mn} + \beta_n u_{3mn}) = \lambda_{mn}^2 v_{1mn} + \frac{\partial}{\partial x_1}(\alpha_m v_{2mn} + \beta_n v_{3mn}) = \lambda_{mn}^2 v_{1mn} + \frac{\partial R_{mn}}{\partial x_1} = \Omega_{vmn}$$

and using (52)-(54) we obtain

$$\mu \Omega_{vmn}|_{x_1=0} = \alpha_m S_{1mn} + \beta_n T_{1mn} + 2\mu\lambda_{mn}^2 \delta_{1mn}, \tag{58}$$

$$\mu \Omega_{vmn}|_{x_1=a} = \alpha_m S_{2mn} + \beta_n T_{2mn} + 2\mu\lambda_{mn}^2 \delta_{2mn}. \tag{59}$$

Substituting (44) into (55) and (59) we find $c_{1mn}(t)$ and $c_{2mn}(t)$. The conditions (52) give, using (38) and (55),

$$\left[v_{1mn} + \gamma_1 \int_0^t q_{1mn}(\tau)\,d\tau + \frac{\partial F_{mn}}{\partial x_1}\right]\Bigg|_{x_1=0} = \delta_{1mn}(t), \tag{60}$$

$$\left[v_{1mn} + \gamma_1 \int_0^t q_{2mn}(\tau)\,d\tau + \frac{\partial F_{mn}}{\partial x_1}\right]\Bigg|_{x_1=a} = \delta_{2mn}(t). \tag{61}$$

Substituting (47) into (60) and (61) and noting that $c_{1mn}(t)$ and $c_{2mn}(t)$ are now known, we find $c_{3mn}(t)$ and $c_{4mn}(t)$. We define a new function $\tilde{\theta}_{mn}(x_1, t)$ by

$$\theta_{mn}(x_1, t) = \tilde{\theta}_{mn}(x_1, t)\exp(-k^2\lambda_{mn}^2 t) + q_{1mn}(t)\,x_1 + \frac{x_1^2}{2a}\,[q_{2mn}(t) - q_{1mn}(t)]. \tag{62}$$

Substituting (62) into (41) and (55) we obtain

$$\dot{\tilde{\theta}}_{mn} = k^2\frac{\partial^2\tilde{\theta}_{mn}}{\partial x_1^2} + \tilde{M}_{mn} - \eta\varkappa\exp(k^2\lambda_{mn}^2 t)\,\dot{e}_{vmn}, \tag{63}$$

$$\left.\frac{\partial\tilde{\theta}_{mn}}{\partial x_1}\right|_{x_1=0} = 0,\quad \left.\frac{\partial\tilde{\theta}_{mn}}{\partial x_1}\right|_{x_1=a} = 0,\quad \tilde{\theta}_{mn}|_{t=0} = \tilde{\theta}_{mn}^0, \tag{64}$$

where $\tilde{M}_{mn}$, $\tilde{\theta}^0_{mn}$ are known, and e_{vmn} have been found by us. The solution of the problem (63) and (64) is known [8]. The quantities $u_{imn}(x_1, t)$ $(i = 1, 2, 3)$ can now be found from (38)-(40) by using (47), (50), and (51). The problem is therefore solved.

Problem 1b. Suppose the boundary conditions have the form (21), (22), (25), (26), (29), and (30). The unknown functions will be expanded in a Fourier series of the form

$$u_1 = \sum_{m,n=0}^{+\infty} u_{1mn}^{(1)}(x_2, t)\sin\gamma_m x_1\cos\beta_n x_3,$$

$$u_2 = \sum_{m,n=0}^{+\infty} u_{2mn}^{(1)}(x_2, t)\cos\gamma_m x_1\cos\beta_n x_3,$$

$$u_3 = \sum_{m,n=0}^{+\infty} u_{3mn}^{(1)}(x_2, t)\cos\gamma_m x_1\sin\beta_n x_3,$$

$$\theta = \sum_{m,n=0}^{+\infty} \theta_{mn}^{(1)}(x_2, t)\cos\gamma_m x_1\cos\beta_n x_3,$$

where $\gamma_m \equiv \pi m/a$ and $\beta_n \equiv \pi n/c$. The corresponding expansions for other functions can easily be written down. Conditions (25), (26), (29), and (30) are satisfied. The solution of the problem can now be solved in the same way as Problem 1a.

Problem 1c. Suppose the boundary conditions have the form (23)-(28). The unknown functions will be expanded in a Fourier series of the form

$$u_1 = \sum_{m,n=0}^{+\infty} u_{1mn}^{(2)}(x_3, t)\cos\alpha_m x_2\sin\xi_n x_1,\quad u_2 = \sum_{m,n=0}^{+\infty} u_{2mn}^{(2)}(x_3, t)\sin\alpha_m x_2\cos\xi_n x_1,$$

$$u_3 = \sum_{m,n=0}^{+\infty} u_{3mn}^{(2)}(x_3, t)\cos\alpha_m x_2\cos\xi_n x_1,\quad \theta = \sum_{m,n=0}^{+\infty} \theta_{mn}^{(2)}(x_3, t)\cos\alpha_m x_2\cos\xi_n x_1,$$

where $\alpha_m \equiv \pi m/b$ and $\xi_n \equiv \pi n/a$. The corresponding expressions for the other functions can easily be written down. Conditions (25)-(28) are satisfied. The problem can now be solved in the same way as Problem 1a. The solution of the general problem 1 with boundary conditions (19)-(24) will now be obtained as a sum of the solutions of the problems 1a, 1b, and 1c.

NOTATION

u_i, displacement vector; $\theta = T - T_0$, temperature drop (the difference between the instantaneous temperature T and the equilibrium temperature T_0); λ and μ, Lamé constants; λ^0, thermal conductivity, c_ε, heat capacity at constant deformation; w, heat generated per unit volume per unit time; α_0, heat expansion coefficient; X_i, volume forces; G, potential of the volume forces; $f_i(x_k)$ and $h(x_k)$, given functions of coordinates x_k $(k = 1, 2, 3)$; t, time; σ_{ij}, components of the stress tensor; a, b, and c, lengths of the edges of the parallelepiped; δ_i, S_i, T_i, and q_i, given functions; for i = 1, 2 they are functions of x_2, x_3, and t, for i = 3, 4 they are functions of x_1, x_3, and t, and for i = 5, 6 they are functions of x_1, x_2, and t. The quantities $c_{imn}(t)$ are unknown functions of time.

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